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The Brightness of the Sky in the Vicinity of the Sun

by

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Solar Department of Harvard College Observatory
Harvard University
Cambridge 38, Massachusetts

(15) Contract No. AF19(604)-4962
Project No. 7649
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GEOPHYSICS RESEARCH DIRECTORATE
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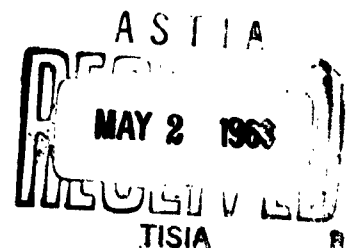
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CONSIGLIO NAZIONALE DELLE RICERCHE

D. H. MENZEL & H. K. SEN

The brightness of the sky in the vicinity of the sun

R O M A

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② **The brightness of the sky in the vicinity of the sun,**

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The study reported on in this paper was carried out originally some ten years ago, during the early planning for Sacramento Peak Observatory. After the site for the coronagraph had been selected, we found it necessary to set up standards of sky brightness, to apply to conditions at the peak. The Government, in acquiring the land, wished to impose restrictions in the region, to prevent some rancher or some industry from producing smoke or dust in quantities sufficient to damage the existing good sky quality. In effect, what we needed was a simple but precise definition of sky quality in terms of potential atmospheric pollution from sources at different distances from the peak. The definition had to be precise, in order to be enforced; it had to be clear if we were to obtain voluntary « easements » from neighboring land owners, agreeing that they would not create a smoke or dust « nuisance ».

We did not receive much help from the definitions of smoke pollution by municipal and such public authorities. For instance, the Public Health Act (1875) of England supplies the standard of smoke in the following terms, « Any chimney

not being the chimney of a private dwelling-house sending forth black smoke in such quantity as to be a nuisance shall be deemed to be a nuisance liable to be dealt with summarily under the Act ». Similar provisions obtain in the United States.

As not more than about 1/4 to 1 per cent of the fuel comes out as smoke [1], the financial loss to the manufacturers is little compared to the inconvenience to the public. The progress of smoke abatement has therefore rested mostly with the arousing of the public conscience.

It is imperative in our case to arrive at a clear definition of what constitutes smoke and dust nuisance, so that it may be enforceable in a court of law. An attempt has been made in this report to frame such a definition. It must be emphasized that no strict solution has been possible on account of the extreme complexity of the problem. The best estimate, however, has been made of the order of magnitude so far as the circumstances of the case did permit. In view of this, any factory that operates at a given distance should be required to use, as far as possible, non-volatile fuel (coke, semi-coke, anthracite, gas,

electricity), the best modern and most nearly smokeless methods of combustion, and an efficient indicator for the density of smoke emitted within a given interval.

As a good deal of relevant literature had to be consulted before a satisfactory solution could be reached, it has been thought appropriate to give, in a supplementary report, a brief review of the smoke problem and a bibliography.

The following are the constituents of of coal smoke:

- (1) Carbonaceous matter - derived from the incomplete combustion of the volatile part of the coal.
- (2) Tar-formed in the process of destructive distillation of bituminous coal, and condensed into smoke.
- (3) Ash - drawn up from the fire by the rush of air passing through the fuel.
- (4) Steam - formed from the hydrogen and the water in the coal.
- (5) Sulphur - derived from sulphides in the coal. It exists originally with water vapour in smoke as sulphur dioxide (SO_2), which becomes oxidized into sulphur trioxide (SO_3) and forms sulphuric acid.
- (6) Grit - particles of unburnt coal or cinder carried away in the draught.

Domestic chimneys, owing to the bad firing conditions, give out more tar, where as factory chimneys give out more ash carried by the draft. The following is a representative analysis [2] of soot from domestic and factory chimneys:

Soot from top of domestic chimney

Carbonaceous matter	55.63 per cent
Tar	37.14 per cent
Ash	7.23 per cent

Soot from top of factory chimney

Carbonaceous matter	48.21 per cent
Tar	0.67 per cent
Ash	51.12 per cent

An analysis of the tar content of the soot would therefore give us a fair estimate of its domestic or factory origin.

The following Tab. 1 [3] gives the values of the rate of settling for spherical particles of unit density, falling in still air.

It is evident from the above table that only particles more than 10^{-3} cm diameter would effectively settle. Smaller particles settle so slowly that they are carried

TABLE. 1 - Rate of settling for spherical particles of unit density, falling in still air.

DIAMETER OF PARTICLE cm	RATE OF SETTLING cm s
10^{-2}	30
10^{-3}	0.3
10^{-4}	0.003
10^{-5}	0.00003

up again by the least upward movement of the air. Observation also indicates [4] the existence of particles of about a micron (10^{-4} cm) in diameter at a sufficient altitude. *We would therefore restrict ourselves to such particles.*

We reproduce below in brief the mathematical analysis of the problem.

Let I be the ratio of the radiation scattered in a unit solid angle close to the direction of incidence to the radiation incident on the geometrical cross-section of the particle.

The scattering efficiency of particles of different sizes are given by the following formulae [5]:

(a) Particles of small size [$r < 1/10 \lambda$ (wavelength of light) $\sim 5 \times 10^{-6}$ cm]

Assume that $x = \frac{2\pi r}{\lambda} < 1/2$.

Rayleigh's formula is appropriate in this case.

$$\pi I = x^4 \frac{(\mu^2 - 1)^2}{(\mu^2 + 2)},$$

where μ = refractive index of particle.

For $\mu = 1.5$, $\pi I = 1/3 x^4$.

(b) Particles of intermediate size ($r \sim 10^{-5}$ cm)

$$\xi = \frac{2\pi r}{\lambda} = 1.2, \text{ and } \pi I = 1.$$

(c) Particles large with respect to wavelength of light (λ).

$$\xi = \frac{2\pi r}{\lambda} = 1.2 \times 10^6 r.$$

πI (central diffraction peak close to sun's limb) $= 1/4 \xi^2 = 0.36 \times 10^{10} r^2$.

For $r = 10^{-4}$ cm, $\xi = 12$, $\pi I = 36$;

for $r = 10^{-3}$ cm, $\xi = 120$, $\pi I = 3600$.

We shall use the following formula for the surface brightness, H , of the sky near the sun in terms of the surface brightness H_\odot of the sun:

$$(1) \quad \frac{H}{H_\odot} = \pi I \sec \zeta \int_{h_0}^{\infty} n \, dh \cdot \omega_\odot r^2,$$

where πI is as defined above.

ζ = zenith distance of the sun;

ω_\odot = solid angle in radians subtended by the sun at the observer $= 0.4 \times 10^{-4}$;

r = radius of particle;

n = number of particles/cm³;

h_0 = altitude of station above source of pollution.

The space distribution of particles from a given source will depend on the meteorological conditions. SUTTON [6] gives formulae for the space distribution of particles given off by a point source continuously in terms of the parameters:

Q , number of particles given off per second;

u , wind speed in m/s;

x , distance downwind from the source in metres;

η , a parameter characterizing the stability. By integrating over the vertical from 0 to ∞ , one can get from his formulae the total number of particles in a given path traversed by sunlight. The result given here for a square metre path is

$$(2) \quad N = 2.69 \frac{Q}{xpu}$$

where

N = total number of particles in a vertical path 1 square metre in cross-section;

Q = number of particles given off each second by the source;

u = wind speed in m/s;

x = distance downwind in metres;

p = (3/4 for strong inversion,
 $u \sim 2$ m/s,

(7/8 for adiabatic lapse rate,
 $u \sim 5$ m/s,

(9/10 for convection,
 $u \sim 7$ m/s

We shall make a few remarks on formula (2). There are no observational results which strictly apply to our case. If the problem must be solved theoretically, there is no reason why formula (2) should not be used. It is probably as clearcut an application to our problem as is to be found in the literature. The theory has been tested a few times against observa-

tions and seems to hold within a factor of two or so. Unfortunately, these tests have been made only for concentrations near the ground, and there is no test for higher elevations.

There are grounds for disagreeing with Sutton's basic theory. This is mainly because he assumes that diffusion and wind speed can be represented by a simple power law in the vertical, and observations have shown that this is not true. But for present purposes, the method should be all right as to order of magnitude.

Wherever possible, in deriving the formula for N from Sutton's formulae, the worst possible conditions have been assumed. Thus, the source of the pollution is supposed to be at the ground rather than a high chimney. Also, the formula holds for a point directly downwind where the maximum pollution occurs, and the integration is taken from the ground and makes no allowance for the height of the telescope.

Because of the very different terrain at Sacramento Peak compared with the level country in which Sutton's formulae were tested, there is still some doubt as to their applicability. Nevertheless, the formula (2) may be used for approximate estimates. Probably no better estimate could be obtained without setting up smoke sources in the vicinity of Sacramento

Peak and making direct measurements, which would be, by far, the best way to solve the problem, but not the easiest.

We shall now apply formulae (1) and (2) to the data of our problem. We shall assume

The sun's zenith distance $\zeta = 30^\circ$,
 $\sec \zeta \sim 1$;

Particle-radius = 1 micron = 10^{-4} cm,
 and the corresponding value of $\pi I = 36$.

Eliminating N between formulae (1) and (2) we have

$$(3) \log Q = \frac{3}{4} \log x + \log \frac{H}{H_\odot} + 16.76,$$

where Q is the number of particles emitted by the source in one second, of radius of the order of a micron; x is the distance of the observer from the source in kilometers; and H/H_\odot is the surface brightness of the sky near the sun in terms of the sun's surface brightness.

The following tables are calculated from formula (3) for three values of H/H_\odot , viz.,

ten-millionths, the optimum conditions attainable at present at the Observatory;
 thirty-millionths, limit of coronagraphic observations of the very highest quality;

one-hundred-millionths, limit of visibility of fainter coronal features [7].

TABLE 2 - $H/H_\odot = 10^{-5}$

x	1/2	1	2	3	4	5	6	7	8	9	10
$\log Q$	11.53	11.76	11.99	12.12	12.21	12.28	12.34	12.39	12.44	12.48	12.51
x	11	12	13	14	15	16	17	18	19	20	
$\log Q$	12.54	12.57	12.60	12.62	12.64	12.66	12.68	12.70	12.72	12.74	

TABLE 3 - $H/H_\odot = 3 \times 10^{-5}$

x	1/2	1	2	3	4	5	6	7	8	9	10
$\log Q$	12.01	12.24	12.47	12.60	12.69	12.76	12.82	12.87	12.92	12.96	12.99
x	11	12	13	14	15	16	17	18	19	20	
$\log Q$	13.02	13.05	13.08	13.10	13.12	13.14	13.16	13.18	13.20	13.22	

TABLE 4 - $H/H_{\odot} = 10^{-4}$

x	1/2	1	2	3	4	5	6	7	8	9	10
$\log P$	12.53	12.76	12.99	13.12	13.21	13.28	13.34	13.39	13.44	13.48	13.51
x	11	12	13	14	15	16	17	18	19	20	
$\log Q$	13.54	13.57	13.60	13.62	13.64	13.66	13.68	13.70	13.72	13.74	

Tables 2, 3, and 4 are plotted in Fig. 1.

In view of the very approximate nature of the analysis we recommended that a general clause be included in the contract that any atmospheric pollution will be

second as read from Fig. 1. Obviously the increase in sky-brightness would be additive for more than one factory at different distances from the station, and corresponding modifications should be made in the stipulation. The estimation of the number of particles emitted per second must be made by an expert agreed to by both parties. The plans submitted should in all cases be checked to see that they allow for the use, as far as possible, of non-volatile fuel and the best modern methods of combustion. It is always a great help to have an efficient indicator for the density of smoke emitted within a given interval.

The problem is more complicated for a domestic chimney, as these are spread over a wide area and do not act like a point-source. However, as a rough approximation, a power-factory burning 1000 tons of coal per day may be taken to be 70000 times as powerful a smoke emitter as a domestic chimney [8].

The estimation of the number of particles to be emitted per second by a factory situated at a given distance will, from the nature of the case, rest finally with the heat-engineer. However, as an illustration of the present analysis, we shall estimate the distance, x , for a power factory burning, say, 1000 tons of coal per day. On the assumption that one per cent of the fuel comes out as smoke [1], the rate of emission of smoke will be 100 gms. per second. We may take the average density of the particles, $\rho = 3$. Then, for particles of 1 micron ($= 10^{-4}$ cm.) radius,

$$Q = \frac{100}{\frac{4}{3} \pi \times 10^{-12} \times 3} = 10^{13}.$$

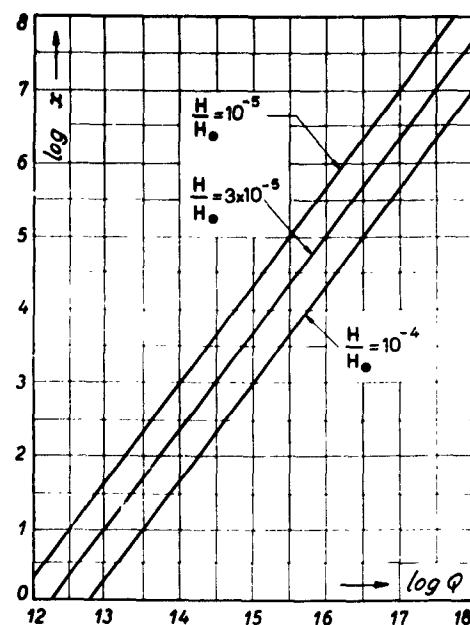


FIG. 1.

deemed a nuisance which increases the sky-brightness near the sun (say, at about 9 minutes of height from the sun's limb) as measured from the observing station beyond a certain optimum limit (say, thirty-millionths of that of the sun).

The general clause should be supplemented by the stipulation that any factory proposed to be built at a distance x (in kilometres) from the observing station must not emit more than Q particles per

From Fig. 11, we read the following values for x (in kilometres):

$H/H_0 =$	$3 \cdot 10^{-4}$	10^{-4}
$x =$	10	2

Thus, the factory should preferably be situated 10 kilometres from the station and, in any case, not nearer than 2 kilometres.

In conclusion, the authors wish to express thanks to Dr. van de Hulst and to Dr. Richard A. Craig for considerable help in the mathematical analysis.

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BIBLIOGRAPHICAL REFERENCES

- [1] OWENS: "Smoke Problem of Great Cities", p. 226.
- [2] OWENS: *loc. cit.*, p. 34.
- [3] GIBBS: "Clouds and Smokes", p. 129.
- [4] ALLCUT: in *Eng. Sci.*, **20**, 154, (1947).
- [5] VAN DE HULST: in *Thesis*, Utrecht, (1946).
- [6] *Quarterly Journal Roy. Meteor. Soc.*, **73**, 426, (1947).
- [7] HIGH ALTITUDE OBSERVATORY, Climax, Colorado, *Special Report*, July 10, 1948, p. 20.
- [8] *Journal, Institute of Fuel*, **8**, Feb. 1935, p. 120.

SUMMARY

A study made of the scattering of small particles in the earth's atmosphere set a standard for studies of atmospheric pollution by dust, smoke, or other particles. The study takes into account the diffusion of particles from a source of pollution, such as an industrial chimney, and sets limits for the pollution in terms of sky brightness immediately surrounding the sun. This study was first undertaken in connection with the establishment of the Sacramento Peak Observatory, in New Mexico.